Spring Semester 2005

Topics in Computation Theory (CS700)

Discrete Geometry

Homework 7

This homework is due on *Tuesday* **June 14**, at the beginning of the class at 1:00 p.m.

On the top of the first sheet that you turn in, please put (a) your name and student number, (b) how much time you spent working on the homework, and (c) a little table with your self-evaluation as explained on the course webpage.

1. Let $(X, \preceq)$ be a finite poset. Prove that if $\preceq$ is not a linear ordering, then there always exist $a, b \in X$ with $|h_{\preceq}(a) - h_{\preceq}(b)| < 1$.

2. Let $g_1, g_2, \ldots, g_m \subset \mathbb{R}^2$ be graphs of piecewise linear functions $\mathbb{R} \to \mathbb{R}$ that together consist of n segments and rays. Prove that the lower envelope of $g_1, g_2, \ldots, g_m$ has complexity $O(\frac{n}{m}\lambda_3(2m))$; in particular, if $m = O(1)$, then the complexity is linear.

3. Given a construction of a set of n segments in the plane with lower envelope of complexity $\sigma(n)$, show that the lower envelope of n triangles in $\mathbb{R}^3$ can have complexity $\Omega(n\sigma(n))$.

4. Let S be a set of n points in the plane, let $s \in S$, and consider the Voronoi cell $V := \{x \in \mathbb{R}^2 \mid d(x, s) = d(x, S)\}$. Show that for any $\varepsilon > 0$ there exists an approximate Voronoi cell V_ε, which is a convex polygon with $O(1/\varepsilon)$ sides such that $V \subset V_\varepsilon$ and $x \in V_\varepsilon$ implies that $d(x, s) \leq (1 + \varepsilon)d(x, S)$.

   Hint: Draw $O(1/\varepsilon)$ rays starting in s, partitioning the plane into $O(1/\varepsilon)$ regions.