

Spring Semester 2005
Topics in Computation Theory (CS700)
Discrete Geometry
Homework 6

This homework is due on *Wednesday May 25*, at the beginning of the (extra) class at 4:00 p.m.

On the top of the first sheet that you turn in, please put (a) your name and student number, (b) how much time you spent working on the homework, and (c) a little table with your self-evaluation as explained on the course webpage.

1. Let $I'(m, n)$ be the maximum number of incidences between a set of m points and a set of n unit radius circles. Use the crossing lemma to prove that $I'(n, n) = O(n^{4/3})$. (The difficulty is handling multiple edges in the graph you define.)
2. Prove that $I'(m, n) = O(m\sqrt{n} + n)$ using the method of Lemma 4.5.1 from the book.
3. Prove that in a drawing of G with the smallest possible number of crossings, no two edges cross more than once.
4. Explain why the previous result does not imply that, given a graph G , the minimum number of pairs of crossing edges in a drawing of G is $\text{cr}(G)$.
5. Let L be a set of n lines in the plane, and let g be a line not in L . Prove that the total complexity of all cells of the arrangement of L that are intersected by the line g is $O(n)$, using the fact that $\lambda_2(n) = O(n)$. (Hint: Assume g to be horizontal. Consider only the part *above* g , and assign one symbol to each side of each line.)