Part I

Baseball Pennant Race

Pennant Race

TUESDAY, SEPTEMBER 10, 1996

San Francisco Chronicle

The Gate
Sports Online
http://www.sfgate.com

SPORTING G

49ers, Young Get Big Brea

Quarterback m

By Gary Swan
Chronicle Staff Writer

The bye week has come at a perfect time for the 49ers and quarterback Steve Young. If they had a game next Sunday, there's a good chance Young would not play.

But the pulled groin muscle on his up-

Giants Officially Leave the NL West Race

By Nancy Gay
Chronicle Staff Writer

CARDINALS 6
GIANTS 2

With the smack of another National League West bat 500 miles away, the Giants' run at the division title ended last night, just as they were handing the visiting St. Louis Cardinals an even bigger lead in the NL Central.

In San Diego, Greg Vaughn's three-run homer in the eighth pushed the Padres over the Pirates and officially shoved the rest of the Giants' season into the background. On the heels of their tedious 6-2 loss before an announced crowd of 10,307 at Candlestick Park, the Giants fell 19½ games off the lead.

As it is, the worst the Padres (80-65) can finish is 80-82. The Giants have fallen to 59-83 with 20 games left; they cannot win 80 games. Coming off a miserable 2-8 mark on a three-city road trip that saw their road record drop to 27-47, the Giants were hoping to get off on the right foot in their longest homestand of the year (15 games, 14 days).

Financing in Place For Giants' New Stadium
SEE PAGE B1, MAIN NEWS

"Where we are, you're going to be eliminated sooner or later," Baker said quietly. "But it doesn't alter the fact that we've still got to play ball. You've still got to play hard, the fans come out to watch you play. You've got to play for the fact of loving to play, no matter where you are in the standings.

"You've got to play the role of spoiler, to not make it easier on

GIANTS: Page D5 Col. 3

Pennant Race: Example

Example

Team	Won	Left
New York	92	2
Baltimore	91	3
Toronto	91	3
Boston	89	2

Can Boston win the pennant?
No, because Boston can win at most 91 games.

Another Example

Example

Team	Won	Left
New York	92	2
Baltimore	91	3
Toronto	91	3
Boston	90	2

Can Boston win the pennant?
Not clear unless we know what the remaining games are!

Refining the Example

Example

Team	Won	Left	NY	Bal	Tor	Bos
New York	92	2	—	1	1	0
Baltimore	91	3	1	—	1	1
Toronto	91	3	1	1	—	1
Boston	90	2	0	1	1	—

Can Boston win the pennant? Suppose Boston does

1. Boston wins both its games to get 92 wins
2. New York must lose both games; now both Baltimore and Toronto have at least 92
3. Winner of Baltimore-Toronto game has 93 wins!

Abstracting the Problem

Given

1. A set of teams S
2. For each $x \in S$, the current number of wins w_x
3. For any $x, y \in S$, the number of remaining games g_{xy} between x and y
4. A team z

Can z win the pennant?

Towards a Reduction

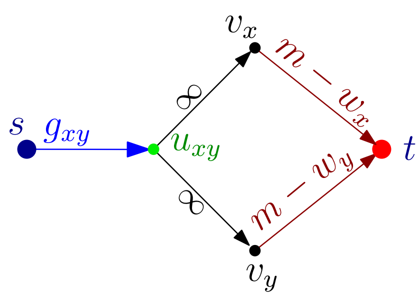
$\overline{z}$ can win the pennant if

1. $\overline{z}$ wins at least m games
   - 1.1 to maximize $\overline{z}$'s chances we make $\overline{z}$ win all its remaining games and hence $m = w_{\overline{z}} + \sum_{x \in S} g_{x\overline{z}}$
2. no other team wins more than m games
   - 2.1 for each $x, y \in S$ the g_{xy} games between them have to be *assigned* to either x or y.
   - 2.2 each team $x \neq \overline{z}$ can win at most $m - w_x - g_{x\overline{z}}$ remaining games

Is there an assignment of remaining games to teams such that no team $x \neq \overline{z}$ wins more than $m - w_x$ games?

Flow Network: The basic gadget

1. s: source
2. t: sink
3. x, y: two teams
4. g_{xy}: number of games remaining between x and y.
5. w_x: number of points x has.
6. m: maximum number of points x can win before team of interest is eliminated.

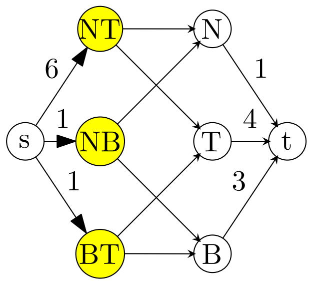


Flow Network: An Example

Can Boston win?

Team	Won	Left	NY	Bal	Tor	Bos
New York	90	11	—	1	6	4
Baltimore	88	6	1	—	1	4
Toronto	87	11	6	1	—	4
Boston	79	12	4	4	4	—

1. $m = 79 + 12 = 91$: Boston can get at most **91** points.



Constructing Flow Network

Notations

1. S: set of teams,
2. w_x wins for each team, and
3. g_{xy} games left between x and y.
4. m be the maximum number of wins for $\overline{z}$,
5. and $S' = S \setminus \{\overline{z}\}$.

Reduction

Construct the flow network G as follows

1. One vertex v_x for each team $x \in S'$, one vertex u_{xy} for each pair of teams x and y in S'
2. A new source vertex s and sink t
3. Edges (u_{xy}, v_x) and (u_{xy}, v_y) of capacity ∞
4. Edges (s, u_{xy}) of capacity g_{xy}
5. Edges (v_x, t) of capacity equal $m - w_x$

Correctness of reduction

Theorem

G' has a maximum flow of value $g^* = \sum_{x,y \in S'} g_{xy}$ if and only if $\overline{z}$ can win the most number of games (including possibly tie with other teams).

Proof of Correctness

Proof.

Existence of g^* flow $\Rightarrow$ $\overline{z}$ can win pennant

1. An integral flow saturating edges out of s, ensures that each remaining game between x and y is added to win total of either x or y
2. Capacity on (v_x, t) edges ensures that no team wins more than m games

Conversely, $\overline{z}$ can win pennant $\Rightarrow$ flow of value g^*

1. Scenario determines flow on edges; if x wins k of the games against y, then flow on (u_{xy}, v_x) edge is k and on (u_{xy}, v_y) edge is $g_{xy} - k$ □

Theorem

Theorem

Suppose that team z has been eliminated. Then there exists a "proof" of this fact of the following form:

The team z can finish with at most m wins.

There is a set of teams $\widehat{S} \subset S$ so that

$$\sum_{s \in \widehat{S}} w_x + \sum_{\{x,y\} \subseteq \widehat{S}} g_{xy} > m \left| \widehat{S} \right| .$$

(And hence one of the teams in $\widehat{S}$ must end with strictly more than m wins.)

Certificate that z cannot win

If z cannot win, then maxflow has value less than g^*.
By max-flow-min-cut theorem, there is a cut (S, T) of capacity $\alpha < g^*$.
Let $\widehat{S}$ be the set of teams x such that $v_x \in \widehat{S}$.

Helper claim

Claim

For any two teams x and y for which the vertex u_{xy} exists, we have that $u_{xy} \in S$ if and only if both x and y are in $\widehat{S}$.

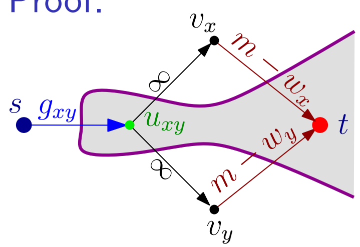
Proof...

$\left(x \notin \widehat{S} \text{ or } y \notin \widehat{S}\right) \implies u_{xy} \notin S$

If x is not in $\widehat{S}$ then v_x is in T. But then, if u_{xy} is in S the edge $(u_{xy} \to v_x)$ is in the cut. However, this edge has infinite capacity, which implies that (S, T) is not a minimum cut.

Helper claim proof continued

Proof.



$x \in \widehat{S}$ and $y \in \widehat{S} \implies u_{xy} \in S$

Assume x and y are in $\widehat{S}$, then v_x and v_y are in S. If $u_{xy} \in T$ then consider the new cut formed by moving u_{xy} to S. For the new cut (S', T') we have

$$c(S', T') = c(S, T) - c\Big((s \to u_{xy})\Big) .$$

□

Proof

There are two type of edges in the cut (S, T): (i) $(v_x \to t)$, for $x \in \widehat{S}$, and (ii) $(s \to u_{xy})$ where at least one of x or y is not in $\widehat{S}$. As such, the capacity of the cut (S, T) is

$$c(S,T) = \sum_{x \in \widehat{S}} (m - w_x) + \sum_{\{x,y\} \not\subseteq \widehat{S}} g_{xy}$$

$$= m\left|\widehat{S}\right| - \sum_{x \in \widehat{S}} w_x + \left(g^* - \sum_{\{x,y\} \subseteq \widehat{S}} g_{xy} \right).$$

However, $c(S,T) < g^*$, and it follows that

$$m\left|\widehat{S}\right| - \sum_{x \in \widehat{S}} w_x - \sum_{\{x,y\} \subseteq \widehat{S}} g_{xy} <= 0.$$