Spring Semester 2005

Topics in Computation Theory (CS700)

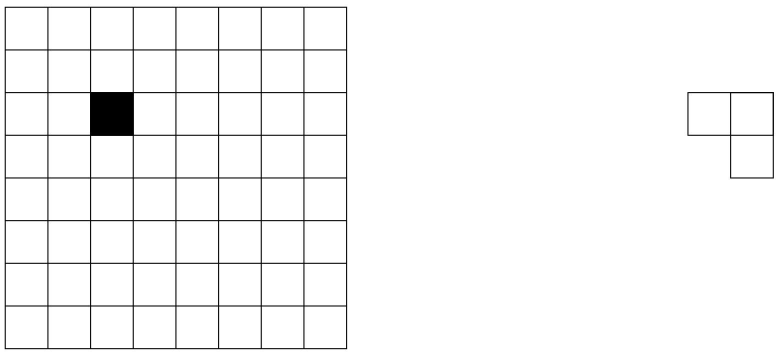
Discrete Geometry

This homework is due on **March 10**, at the beginning of the class.

On the top of the first sheet that you turn in, please put (a) your name and student number, (b) how much time you spent working on the homework, and (c) a little table with your self-evaluation as explained on the course webpage.

Since we haven't covered much material yet, this is a warm-up homework. I'd like to get you into the mood of this course. The last two problems are a bit harder (especially the last one). There is no reason to feel bad if you cannot solve them at this stage — don't spend more than an hour on them.

1. Consider a $2^n \times 2^n$ chess board, from which one square has been removed (any square—we don't know which one). Show that the remaining squares can be covered completely, without overlap or gaps, by copies of an L-shape consisting of three squares as in the following figure (the L-shape may be rotated freely). (The left hand side shows the chess board for $n = 3$, the right hand side shows the L-shape.)



2. Consider a stop of bus 509, which runs every 14 minutes. We assume the stop is so far from the start of the line that buses can be assumed to arrive at a random time. While the expected waiting time at such a bus stop is usually given as 7 minutes, one often feels that one waits longer. Is this a psychological effect, or is there a mathematical reason?

3. Let P be a simple polygonal chain, that is, a piecewise straight curve that does not intersect itself. If we draw a line through each segment of the chain, we obtain an arrangement $\mathcal{A}(P)$ of lines, which we call the *induced arrangement* of P. Show that for any simple line arrangement $\mathcal{L}$ (that is, no two lines are parallel, and no three lines go through a common point), there is always a simple polygonal chain such that $\mathcal{L} = \mathcal{A}(P)$.

4. Prove or disprove: for any simple line arrangement $\mathcal{L}$, there is a *closed* simple polygonal chain (that is, a *simple polygon*) P such that $\mathcal{L} = \mathcal{A}(P)$.