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20	10	20	25	25	100

Student number	2	0						
Name								

Final Exam — Algorithms (CS 500)

June 15, 2015, 9:00–11:45

Instructions:

- This booklet has **three pages** with **five problems** in total. Check that you have all!
- The space provided should be sufficient for your answer. If you need scratch space, use the back side. If you need more space for your answer, you can also use the back side (but *indicate clearly on the front side* that your answer continues on the back).
- This is a **closed book** exam. You are not allowed to consult any book or notes.
- The questions have to be answered in **English**. Write clearly!
- To ensure a quiet exam environment, we will **not answer questions** during the exam. If you think there is a mistake in the question, explain so, and use common sense to answer the question.
- Points will be deducted for solutions which are correct but excessively complicated, hard to understand, or poorly explained. **Keep your solutions short and crisp.**
- Before you start: Write your name and student number (one digit per square!) on **all three pages** of this exam booklet (−5 points for missing names or unreadable numbers).
- Relax. Breathe. This is just an easy, silly, and stupid final.

Problem 1: (20 pts) For each of the following statements, say whether they are *true* or *false* (2 points for correct answer, 0 points for *no answer*, −2 points for *wrong answer*).

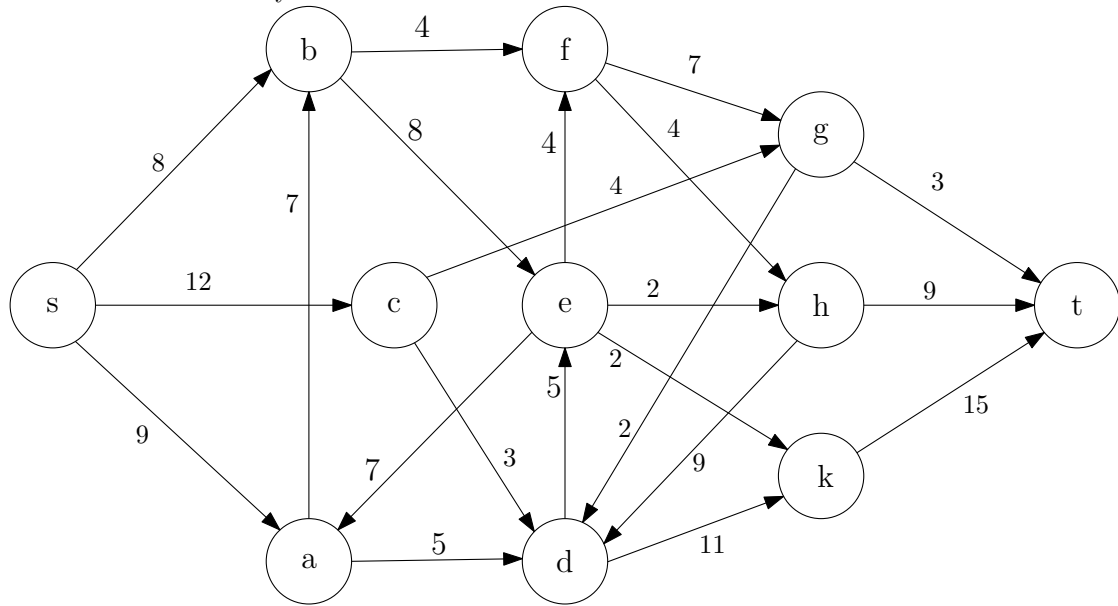
- | | |
|--|--------------|
| 1. To prove that problem X is NP-complete, one needs to prove a reduction $X \leq Y$, where Y is an NP-complete problem. | True — False |
| 2. Whether a linear program is infeasible depends on the cost vector. | True — False |
| 3. In the I/O-efficient model, all computations in main memory are free. | True — False |
| 4. Whether a linear program is unbounded depends on the cost vector. | True — False |
| 5. A linear program with integer coefficients has an optimal solution with integer values. | True — False |
| 6. In the I/O-efficient model, HeapSort is better than MergeSort. | True — False |
| 7. The <i>weak</i> duality theorem can be used to prove that a feasible solution to a linear program is optimal. | True — False |
| 8. Storing a matrix of size 10000×10000 in Z-order can be faster because of fewer cache misses. | True — False |
| 9. A feasible solution for an ILP is also a feasible solution for its relaxation. | True — False |
| 10. A circulation network where every vertex has supply/demand zero, but some edges have positive lower bounds, never has a valid circulation. | True — False |

Problem 2: (10 points) Give the dual of the following linear program.

$$\begin{array}{ll}
 \text{maximize} & x_1 + 2x_2 \\
 \text{such that} & x_1 + 3x_2 \leq 21 \\
 & -x_1 + 3x_2 \leq 18 \\
 & x_1 - x_2 \leq 5 \\
 & x_1, x_2 \geq 0
 \end{array}$$

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Problem 3: (20 points) The figure below shows a flow network, where the number on the edge means the capacity of the edge. Give a minimum-cut for this flow network. What is the capacity of this minimum-cut? Prove that your cut is indeed a minimum-cut!



Problem 4: (25 points) You are given a bipartite graph $G = (U \cup V, E)$, where *every* vertex in the left set U has degree 18, and *every* vertex in the right set V has degree 45.

Prove that there is a subset $H \subseteq E$ of the edges such that in the subgraph $G' = (U \cup V, H)$ every vertex in U has degree two and every vertex in V has degree five.

If you write "I don't know" and nothing else, you will receive 7 points.

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Problem 5: (25 points) Your company makes two kinds of sauces, a *meat sauce* and a *mushroom sauce*. Both are made by mixing three ingredients: minced beef, mushrooms, and tomato concentrate. The composition of the sauces must follow these specifications:

Ingredient	meat sauce	mushroom sauce
beef	at least 40% of total weight	no specification
mushrooms	no specification	at least 30% of total weight
tomato	at most 35% of total weight	at most 50% of total weight

Every day, you can buy 4,000 kg of minced beef for 4,200 Won per kg, 3,200 kg of mushrooms for 1,600 Won per kg, and 6,000 kg of tomato concentrate for 1,400 Won per kg.

The sales price is 8,400 Won per kg for the meat sauce and 7,200 Won per kg for the mushroom sauce.

Assuming that you can sell arbitrary amounts of sauce, write a linear program to compute the daily amounts (and sauce composition) that will maximize your profit. Explain the meaning of your variables and your constraints!

If you write “I don’t know” and nothing else, you will receive 7 points.