

PARTITION \leq LOAD BALANCING

$$X = \{x_1, x_2, \dots, x_n\} \longrightarrow n_1, n_2, t_1, \dots, t_n$$

Is there subset S $t_i = x_i$ $m = 2$

s.t $\sum_{i \in S} x_i = \sum_{i \notin S} x_i$

$$\sum_{i \in S} x_i = \frac{\sum x_i}{2} \iff \max\left(\sum_{i \in S} x_i, \sum_{i \notin S} x_i\right) = \frac{\sum x_i}{2}$$

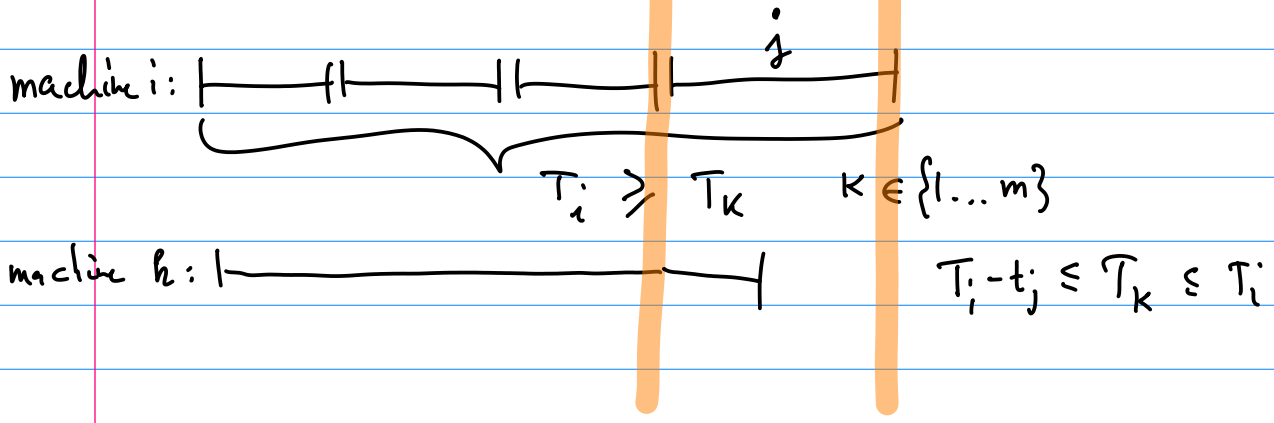
| | | | | |
|---------|---|---|---|-----------------|
| 1, 2, 5 | 7 | } | 7 | $\sum t_i = 19$ |
| 6, 3 | 6 | | | 3 machines |
| 4 | 6 | | | |

$$\text{Opt}(X) \leq \text{Greedy}(X) \leq 2 \text{Opt}(X)$$

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- ① $T^* \geq \max_j t_j$
 - ② $T^* \geq \frac{1}{m} \sum_i t_j$

Consider greedy schedule.

$$T_i = \text{Greedy}(X)$$

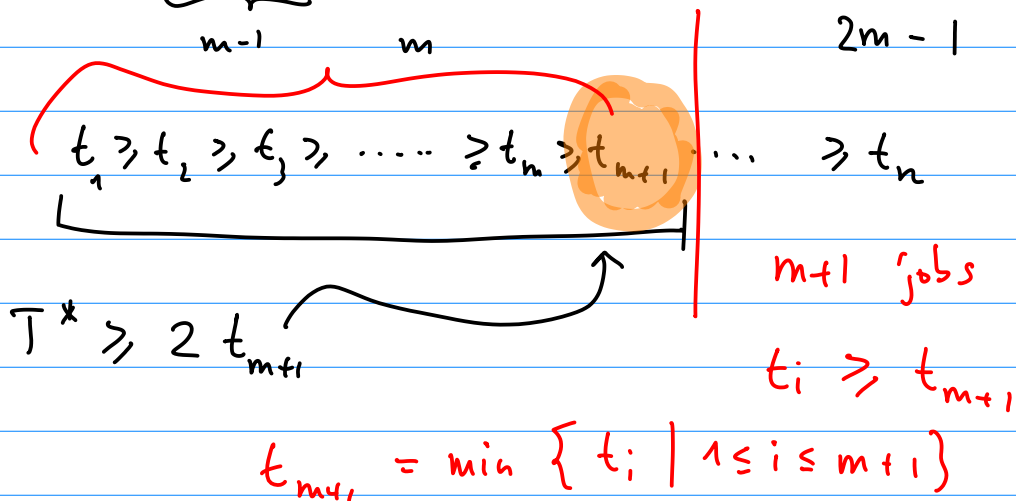
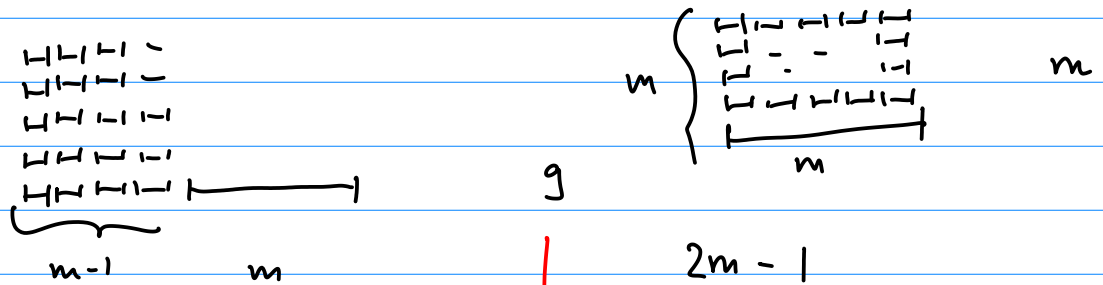


$$\sum_k T_k \geq \sum_k (T_i - t_j) = m(T_i - t_j)$$

$$T_i \leq \underbrace{\frac{1}{m} \left(\sum_k T_k \right)}_{\sum_j t_j} + \underbrace{t_j}_{\leq \max t_j}$$

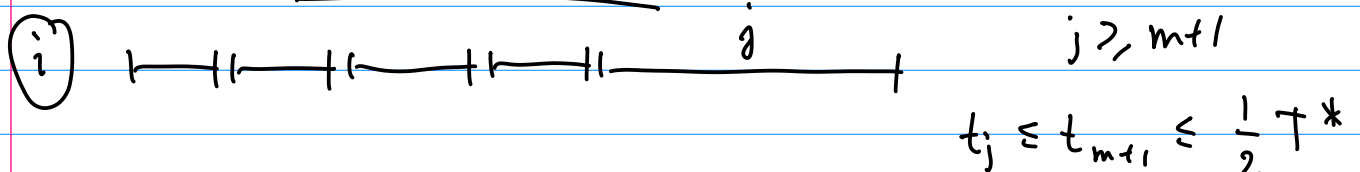
$$T_i \leq \frac{1}{m} \sum_j t_j + \max t_j \leq T^* + T^* = 2T^* \quad \square$$

$m(m-1)$ short jobs $t_i = 1$
 1 long job $t_n = m$



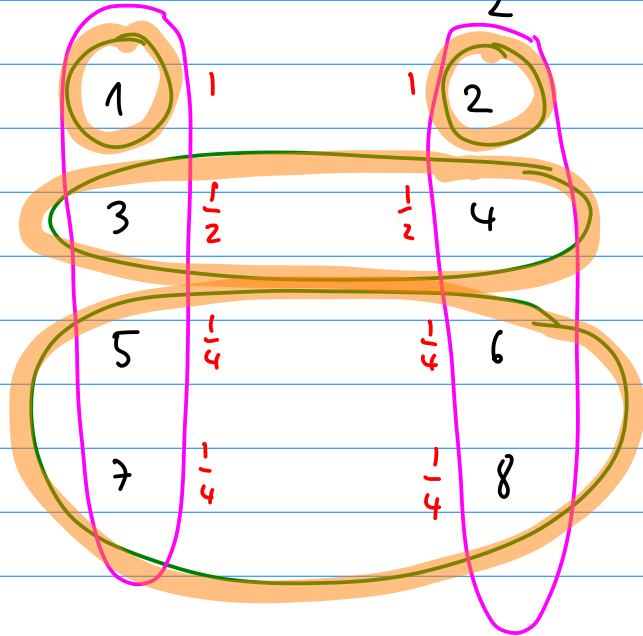
Some machine has at least 2 jobs of length $\geq t_{m+1}$

$$\Rightarrow T^* \geq 2t_{m+1} \quad \square$$



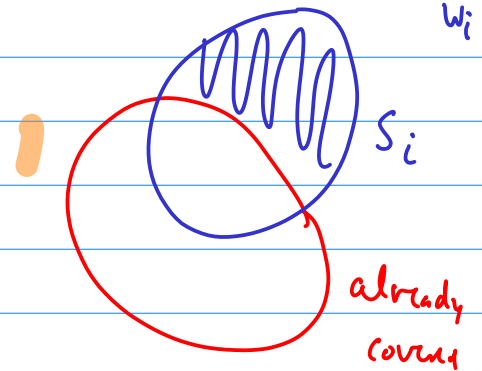
$$T_i \leq \frac{1}{m} \sum_j t_j + t_j$$

$$T_2 \leq T^* + \frac{1}{2}T^* = \frac{3}{2}T^*$$



$w_1 = \text{weight } 1$

$w_2 = \text{weight } 1.0001$



Greedy

$S^1 S^2 \dots S^k$
 $\cdot \quad \uparrow$

is a cover $\bigcup_{1 \leq i \leq k} S^i = U$

$s \in U$

S^i is first to cover s

$$c_s = \frac{w(S^i)}{|S^i \cap R|}$$

charging argument

$$H(n) = \sum_{i=1}^n \frac{1}{i} \approx \ln n \quad n\text{th Harmonic number}$$

$$\sum_{S \in S_k} c_s \leq H(|S_k|) \cdot w_k$$

$$S_k = \{s_1, \dots, s_j, s_d\} \text{ in order of being covered.}$$

$\underbrace{\hspace{10em}}_{\subseteq R} \quad d-j+1$

$$c_{s_j} \leq w_k / (d-j+1)$$

$$\sum_{S \in S_k} c_s \leq w_k \sum_{j=1}^d \frac{1}{d-j+1} = w_k \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{d}\right)$$

$$= H(d) \cdot w_k$$

\mathcal{C} = greedy cover

\mathcal{C}^* = optimal cover

$$w_i \geq \frac{1}{H(d^*)} \sum_{S \in S_i} c_s$$

$$\text{OPT} = w(\mathcal{C}^*) = w^* = \sum_{i \in \mathcal{C}^*} w_i \geq \sum_{i \in \mathcal{C}^*} \frac{1}{H(d^*)} \sum_{S \in S_i} c_s$$

$$H(d^*) \cdot w^* \geq \sum_{i \in \mathcal{C}^*} \sum_{S \in S_i} c_s \geq \sum_{S \in \mathcal{C}} c_s = w(\mathcal{C})$$

$$d^* \leq n$$

$\ln n$ - approx

↑
greedy
cover
weight