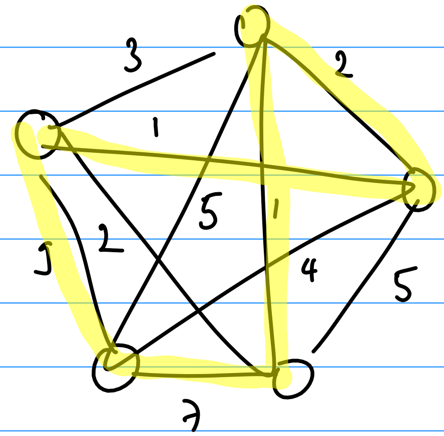
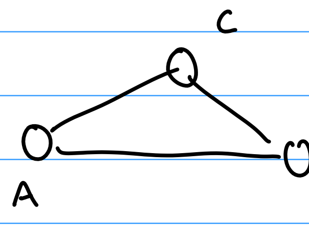
CS 500

April 3, 2015



$n! \approx 2^{n \log n}$



$|AB| \le |AC| + |CB|$

triangle inequality

Is TSP-Min $\in$ NP ? Makes no sense

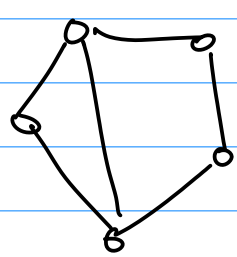
Decision-Version: $G = (V, E)$ $w: E \to \mathbb{R}$

$C > 0$

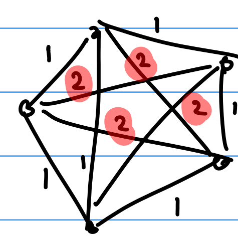
TSP Is there a tour of cost $\le C$?

Is TSP $\in$ NP ?

HamCycle $\to$ TSP



poly-t.



$2 \to c \cdot n$

G has H.C $\iff$ $\underbrace{\text{H has TSP of cost} \le n}$

n edges, cost n

$\downarrow$

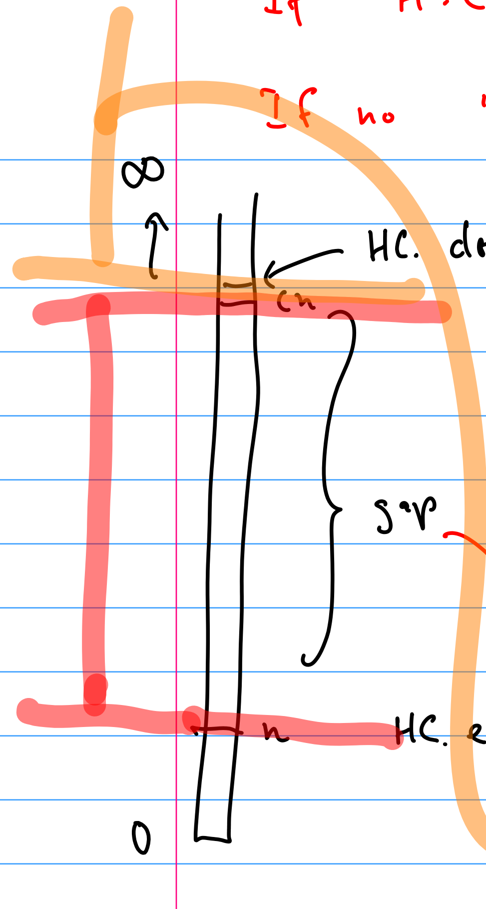
edges have cost 1

$\downarrow$

are in G

If H.C. in G → TSP of cost n

If no " → opt TSP cost $\geq (n-1) + cn$
$\geq cn+1$



c-approx alg A for TSP

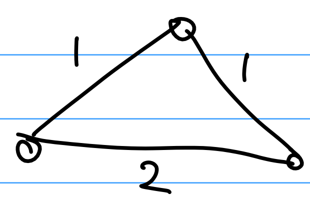
$$OPT(H) \leq A(H) \leq c \cdot OPT(H)$$

1. G has H.C. → $OPT(H) = n$

   $n \leq A(H) \leq c \cdot n$

2. G has no H.C. → $OPT(H) \geq cn+1$

   $cn+1 \leq A(H)$

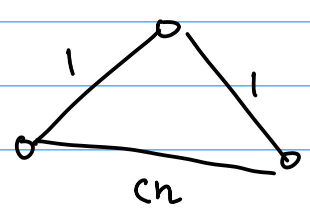


$2 \leq 1+1$ ✓

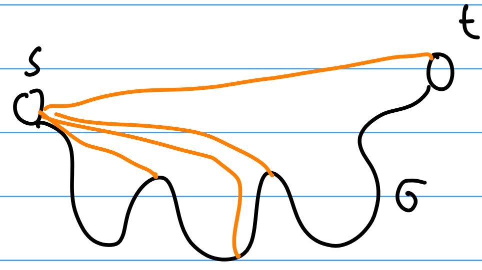
1 1 2 2 1 2

1 2 2

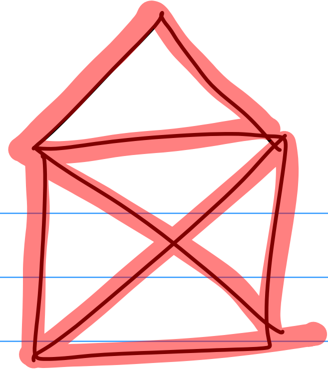
$TSP_{\Delta \neq}$ is still hard.

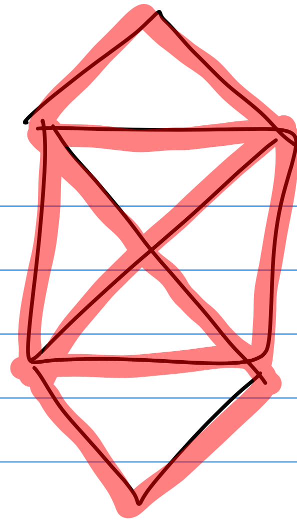


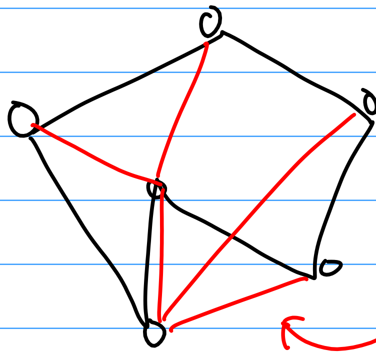
$\cancel{cn \overset{2}{\leq} 1+1}$



$w(s,t) \leq w(\sigma)$





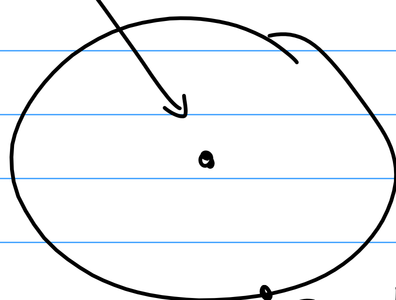


cheapest TSP for G

cheapest spanning graph

MST

TSP



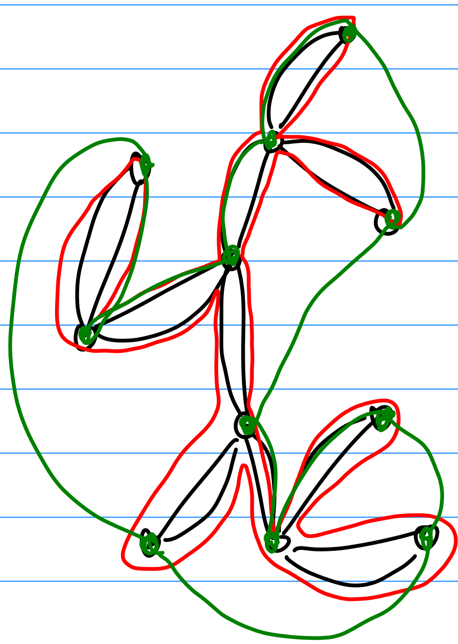
all the spanning graphs

MST

$$A(G) \le c \cdot OPT(G)$$

$$A(G) \le c \cdot w(MST) \le c \cdot OPT(G)$$

$2 \cdot MST$

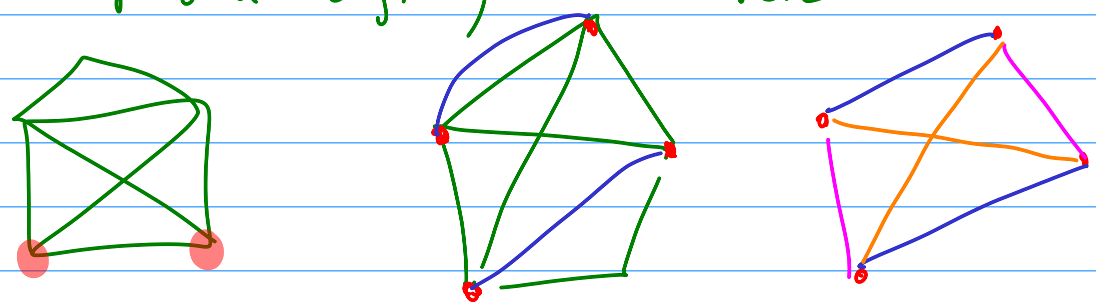


TSP tour

$w(\;) \le 2\,w(MST)$

Remove vertices of odd degree!

(# vertices of odd degree) is even



$$\sum_{v \in V} d(v) = 2|E|$$

$w(\sigma) \leq w(\pi)$ $\quad \Delta \neq$

$w(R) + w(B) = w(\sigma) \leq w(TSP)$

$$\min\{w(R), w(B)\} \leq \frac{w(TSP)}{2}$$

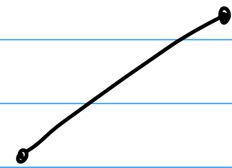
$$w(\text{Min-Weight Perf. Matching}) \leq \min\{w(R), w(B)\} \leq \frac{w(TSP)}{2}$$

$$A(G) \leq \underbrace{w(MST)} + w(MWPM)$$

$$\leq w(OPT) + \frac{w(OPT)}{2} = 1.5\, w(OPT)$$

Euclidean TSP:

Input: Points in $\mathbb{R}^2$



$$w(uv) = d(u,v)$$

Arora $(1+\varepsilon)$-approx