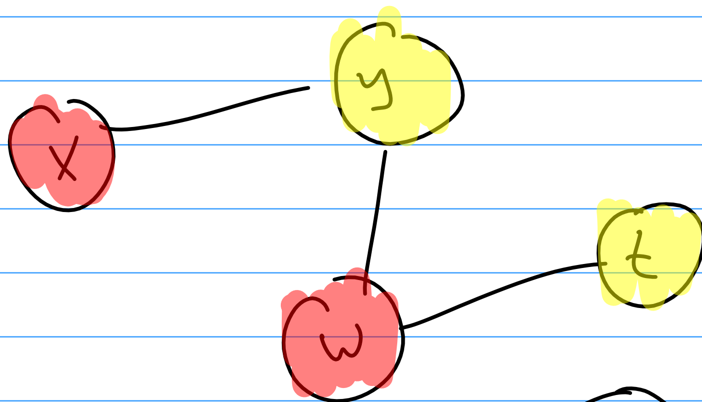
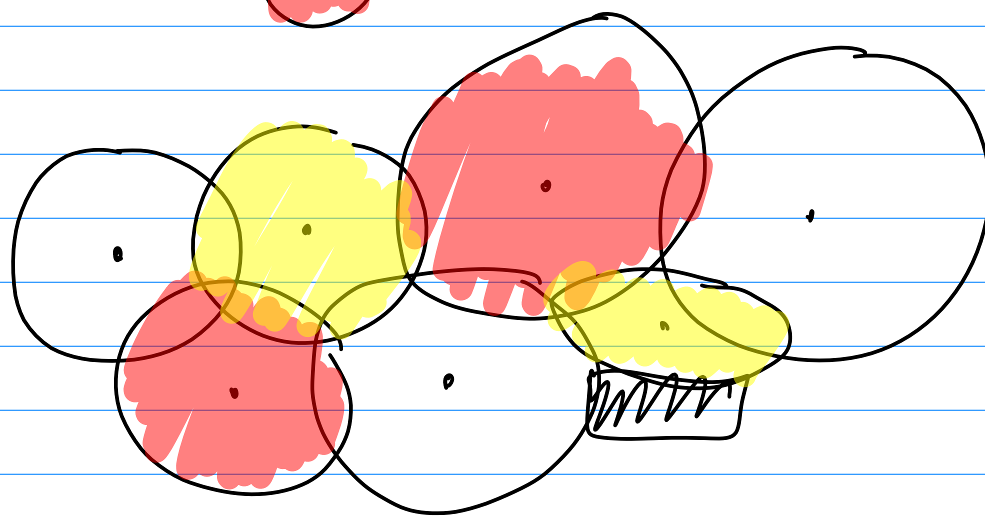
CS 500

April 1st, 2015





$X = \{a_1 \ldots a_n\}$ $\quad B \quad$ Subset Sum

$$\sum_{i \in S} a_i = B$$

KNAPSACK $\quad s_i$ size $\quad$ profit p_i

capacity B $\quad$ target profit P

$$S \qquad \sum_{i \in S} s_i \leq B \qquad \sum_{i \in S} p_i \geqslant P$$

$$I_{SS} = (X, B) \xrightarrow{\text{poly time}} (s_i, p_i, B, P)$$

$\Downarrow$

item $s_i = p_i = a_i$

$P = B$

Subset Sum $\qquad a_1 \; a_2 \; \dots \; a_i \dots \; a_n \qquad B$

Subproblems ;

$S[i,b] =$ using items 1 to i
is there a subset of sum $= b$

$i = 0 \dots n \qquad S[i, 0] = \text{True}$

$b = 0 \dots B \qquad S[0, b] = \text{False} \qquad b > 0$

$$S[i,b] = \begin{cases} S[i-1, b] \vee S[i-1, b-a_i] & \text{if } b \geq a_i \\ S[i-1, b] & \text{else} \end{cases}$$

$i > 0$
$b > 0$

$$(n+1)(B+1) = O(nB)$$

ℓ weeks $\quad L_1, L_2, L_3 .. L_\ell \subseteq \{1, 2, .. n\}$

p projects $\quad P_1, P_2 \dots P_p \subseteq \{1, 2, \dots n\}$

Find NP-complete X and show $X \leq$ LecturePlanning

$X = 3SAT$ $\qquad$ SAT $\leq$ Lecture Planning

$$\phi \xrightarrow{\text{poly-time}} \text{LectPlan}$$

var $x_1 \dots x_n$ $\qquad L_i = \{x_i, \bar{x}_i\}$ $\quad$ 2n speakers

Clauses $C_1 \dots C_m$

$$C_j = x_5 \vee \bar{x}_7 \vee x_9 \Rightarrow P_j = \{x_5, \bar{x}_7, x_9\}$$

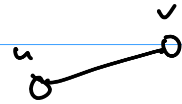
First make choices, later check constraints

VERTEX COVER $\leq$ LectPLANNING

$$G = (V, E),\ k \longrightarrow \begin{matrix} L_i \\ P_j \end{matrix}$$

k weeks $\qquad 1, 2, \dots n \qquad |V| = n$

$\ell = k \qquad L_1 = L_2 = L_3 = \dots = L_k = \{1, 2, \dots n\}$

$e = (u, v)$  $\Rightarrow P_e = \{u, v\}$

one project for each edge

G tree width (G)

small	large
tree	complicated

$2^{tw(G)} \, n^2$

maximization $(1-\varepsilon) \times OPT$

$1.2^{50} \cdot poly$

1 TB 10^{12}