CS 500 3/25

Reductions

Subset Sum:

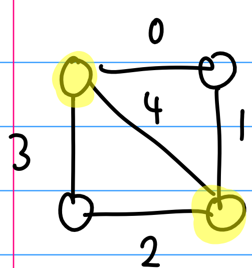
$X = \{3, 19, 25, 37, 14, 2, 9\}$

$t = 48$ Yes

$(G, k) \xrightarrow{\text{poly-time}} (X, t)$

G has VC of size $\le k$ $\iff$ $S \subset X$ $\sum S = t$

$k = 2$



$m = 5$

$X = \{4^0, 4^1, 4^2, 4^3, 4^4,$
$4^5 + 4^0 + 4^3 + 4^4,$
$4^5 + 4^0 + 4^1,$
$4^5 + 4^1 + 4^2 + 4^4,$
$4^5 + 4^2 + 4^3\}$

$t = 2 \cdot 4^5 + 2 \cdot 4^4 + 2 \cdot 4^3 + 2 4^2$
$+ 2 \cdot 4^1 + 2 \cdot 4^0$

Write the numbers in base 4:

X:

1	5 edges
10	
100	
1000	
10000	
111001	← rows for S
100011	
110110	←
101100	
222222	

"⇒" G has VC S

$|S| = k$

if edge i has 2 endpoints in S
⇒ column i already adds
up to 2

for edge i with 1 endpoint in S
use value b_i

⇒ subset sum

"$\Leftarrow$" $S \subset X$ exists with $\sum S = t$

For each edge i, must have $a_v \in S$ for one endpoint of i

But we must have exactly k such a_v in S.

$\{v \mid a_v \in S\}$ is VC of size k. $\square$

"Size" of a_v $\underbrace{1000100000 1000}_{m+1 \text{ digits (base 4)}}$

$2m+2$ bits

$(m+n)(2m+2)$ bits

↑ size of instance of SubsetSum

Subset Sum with Unary Input

$5 = 11111$

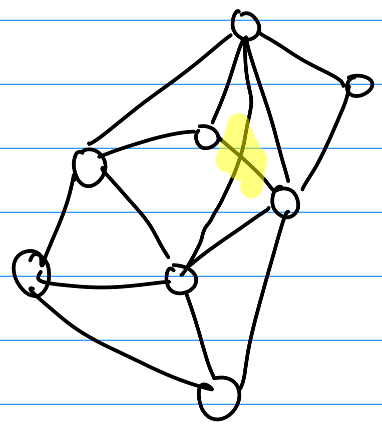
can solve in time $O(n \cdot W)$ ↑ $\sum X$

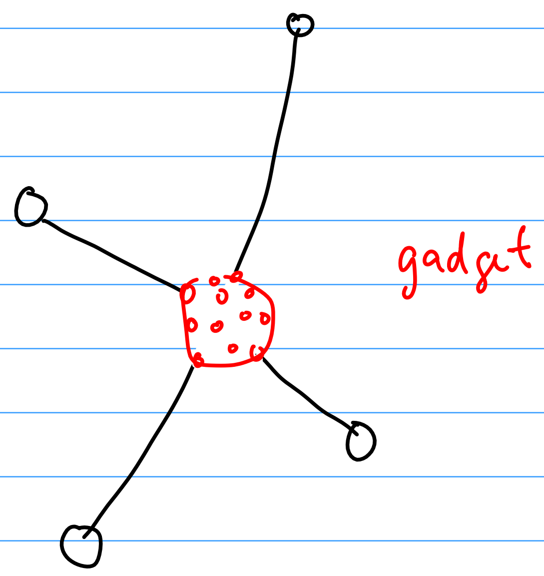
PARTITION: Input X of integers

Q: $S \subset X$ st $\sum S = \sum X \setminus S$

3 COLORING ≤ PLANAR 3 COLORING

G ⟶ G' planar





Assume $x_1, x_2 \dots x_n, u$ s.t. c' is true

Can $c = \ell_1 \vee \ell_2$ be false ? No ! $\Rightarrow$ c is true

SAT $\leq$ 3SAT

2SAT $\leq$ 3SAT

$(x_1 \vee \neg x_2) \wedge (x_2 \vee \neg x_3)$

x_1 false $\Rightarrow$ x_2 false

x_2 true $\Rightarrow$ x_1 true