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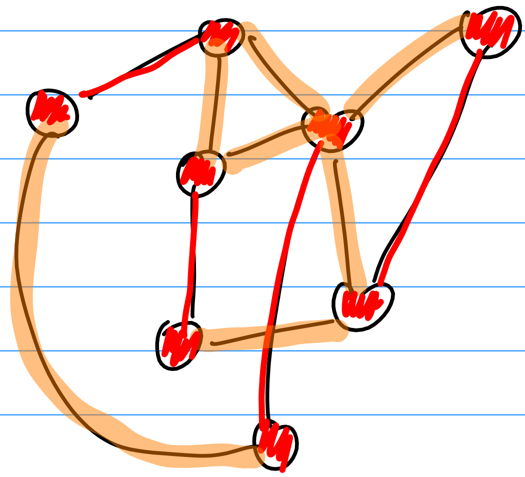
VC: $\quad Opt(X) \le n \qquad \dfrac{A(X)}{Opt(X)} = \ln n$

$A(x) \approx n \ln n$

α-approx alg $\quad A(X) \le \alpha \, Opt(X) \Rightarrow \alpha \ge \ln n$

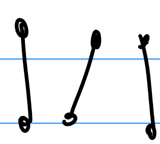


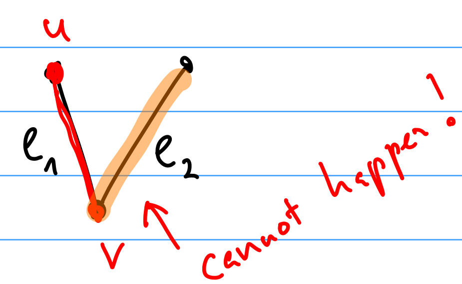
M = edges selected by algorithm

$\boxed{A(X) = 2|M|}$

$A(X) \le 2\, Opt(X)$

M is a matching.





$\boxed{Opt(X) \geq |M|}$

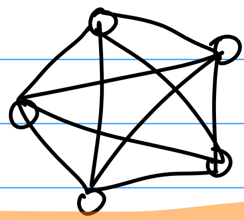
$A(X) = 2|M| \le 2\, Opt(X)$ ☐

How to prove approximation factor:

1. Find lower bound for $Opt(X) \geq L$
2. Prove that $A(X) \le \alpha L$

poly-time reduction $\quad X \leq_p Y$

clique $\cong$ complete subgraph



$\boxed{\begin{array}{l}\text{Reduction:} \quad X \leq Y \\[6pt] \quad I_X \xrightarrow{R} I_Y \\ \qquad - \text{ poly-time} \\ \qquad - \text{ solution is the same}\end{array}}$

G has ind. set of size $= k$

$\Leftrightarrow$ $\underset{\overline{G}}{G'}$ has clique of size $= k' = k$

IndSet $\leq$ Clique $\leq$ IndSet

$n = |I_X| \qquad |I_Y| = p(n)$

R runs in $p(|I_X|) = p(n)$ time

A runs in $q(|I_Y|) = q(p(n))$ time

total running time $\underbrace{p(n) + q(p(n))}_{\text{polynomial!}}$

$X \leq Y \qquad\qquad Y \leq Z$

$$I_X \xrightarrow{\mathcal{R}} I_Y \longrightarrow I_Y \xrightarrow{\mathcal{R}'} I_Z$$

$$p(|I_X|) \qquad\qquad q(|I_Y|)$$

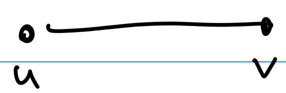
$$I_X \xrightarrow[p(|I_X|) + q(p(|I_X|))]{} I_Z$$

Proof: $G = (V, E) \qquad S \subseteq V$

"$\Rightarrow$" S is V.C. $\Rightarrow \forall e \in E$ some endpoint of e is in S

Pick $u, v \in V \setminus S$

Assume $uv \in E$ $\lightning$



$\lightning$

$\Rightarrow uv \notin E$

$\Rightarrow V \setminus S$ is I.S.

"$\Leftarrow$" $V \setminus S$ is I.S. $\quad \forall u, v \in V \setminus S \Rightarrow uv \notin E$

Take $e = uv \in E$ Assume e is not covered by S $\lightning$

$\Rightarrow u \notin S, v \notin S \Rightarrow u, v \in V \setminus S \Rightarrow uv \notin E$ $\lightning$

$\Rightarrow S$ is a V.C.

$U = \{a, b, c, d, e, f, g\}$ $\qquad k = 2$

$S_1 = \{c, g\}$ $\qquad S_2 = \{b, d\}$ $\qquad S_3 = \{c, d, e\}$

$S_4 = \{e, f\}$ $\qquad S_5 = \{a\}$ $\qquad S_6 = \{a, b, f, g\}$

Vertex Cover $\leq$ Set Cover

$\leq$

Ind Set

$\leq$

Clique