Bonus Slide: Knowing your own Source Code

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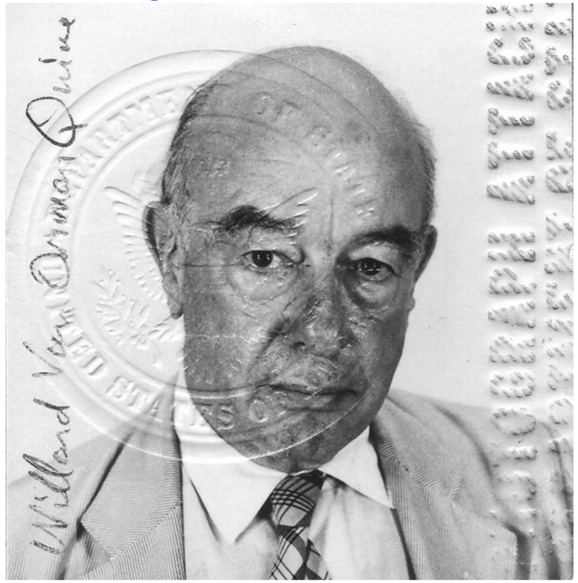
Algorithms and Complexity

SMN Theorem: Given program P, can construct Q that ignores its input and behaves like P on input $\langle P\rangle$.

Q knows source code of P,
but not source code of Q.

Quine Theorem: To any program P of two inputs $(\underline{x},\underline{y})$, there is a program Q on $\underline{x}$ that behaves like P on $(\underline{x},\langle Q\rangle)$.



Example: P prints $\underline{y} \;\Rightarrow\; Q$ prints $\langle Q\rangle$!

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Holds for every Turing-complete programming system!!

Fun: Write a **Quine** in your favorite program. language!

Complexity Theory

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- Problem instance I:
  Is CNF formula $(x \vee y \vee z) \wedge (\neg x \vee \neg y \vee \neg z)$ satisfiable?
- Problem P: <u>in</u>finite collection of instances.
- Algorithmic solution of P: <u>provably</u> solves each $I \in P$
  → model of computation
- Computational complexity $\mathcal{C}_R(P)$:
  minimum amount of resource R consumed by solution
  – worst-case, asymptotically for instance size $|I|=:n \to \infty$
  – resources R: e.g. running time, memory, #processors
- upper complexity bound $\mathcal{C}_R(P) \leq f(n)$:
  <u>exists</u> algorithm solving P <u>provably</u> using at most $f(n)$ R
- lower complexity bound $\mathcal{C}_R(P) \geq g(n)$:
  <u>every</u> algorithm solving P <u>provably</u> uses at least $g(n)$ R

Lower Bounds Techniques

Upper bound: design and analyze an algorithm

$\exists$algorithm A $\forall n$ $\forall$instances I of size $|I|=n$:

A solves I and uses $\leq f(n)$ of R

Lower bound:

$\forall$algorithms A solving P $\forall n$

$\exists$instance I, $|I|=n$: A uses $\geq g(n)$ of R

Proof techniques: a) Diagonalization

b) Show problem P complete for a class (eg. **NP**

– ie. reduce SAT to P in poly.time: algorithm design!)

c) Genius still sought

Morgenstern

– in restricted models: Ben-Or, Razborov, Ukkonen...

Advertisement I:
Space Complexity

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So-far focus on *fast* algorithms, i.e. with low running time

Captured in complexity classes like **P**, **NP**, **EXP**

- and open questions: **P** vs. **NP** vs. **coNP** vs. **EXP**

Often at the expense of memory: Dynamic Programming.

How about *frugal* algorithms: low memory consumption?

Captured in complexity classes like **L**, **NL**, **PSPACE**

- with interesting connections to *parallel* complexity
- and surprising results: $\mathbf{NL}=\mathbf{coNL}\subseteq\mathbf{L}^2\subset\mathbf{PSPACE}$
- and $\mathbf{DTIME}(t(n)) \subseteq \mathbf{DSPACE}(t(n)/\log t(n))$

Advertisement II: Own Research

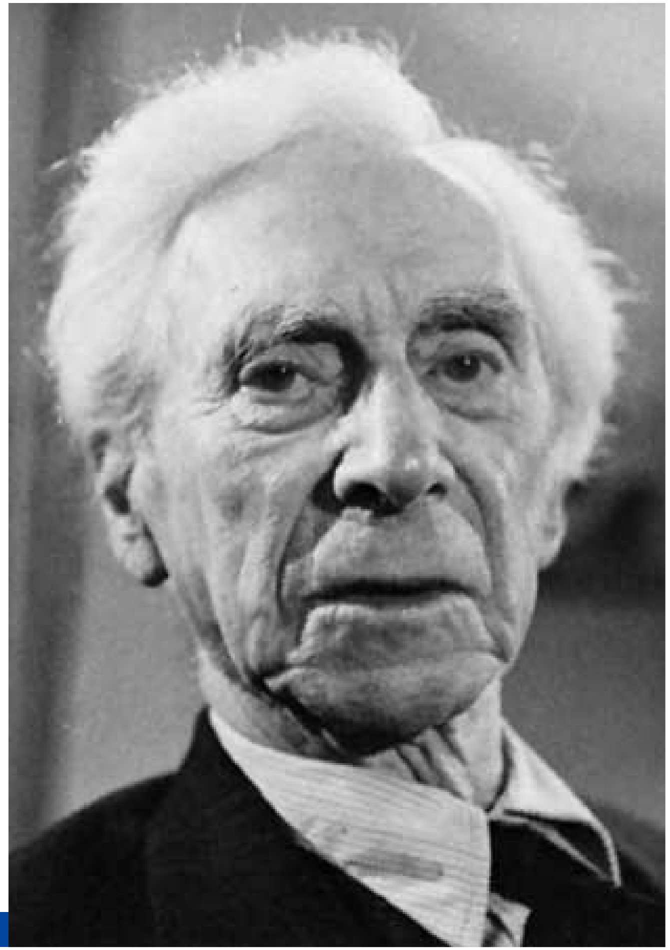
- Computability over real numbers
  - Computing $y \in \mathbb{R}$ means to output an infinite sequence of integer fractions $|q_n - y| \leq 2^{-n}$.

    $y = f(x)$ upon input of a sequence $|p_m - x| \leq 2^{-m}$.
  - Computing $x \to f(x) = y$ means to apply a finite sequence of exact arithmetical operations and comparisons.
- Physical Foundations of Computing and
- Complexity Theory of Computational Physics
- Computational Geometry in Computer Graphics

How Science Works…

Bertrand Russel (1872-1970):

„Darin besteht das Wesen der Wissenschaft:
Zuerst denkt man an etwas, das wahr sein könnte.
Dann sieht man nach, ob es der Fall ist
und im allgemeinen ist es nicht der Fall.“



Extension:

Typically one comes up with an intuitive conjecture, but fails to prove or disprove!

This has to be dealt with,
some way or another…

The End of CS493

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Thanks a lot
for your
attention and
diligence!