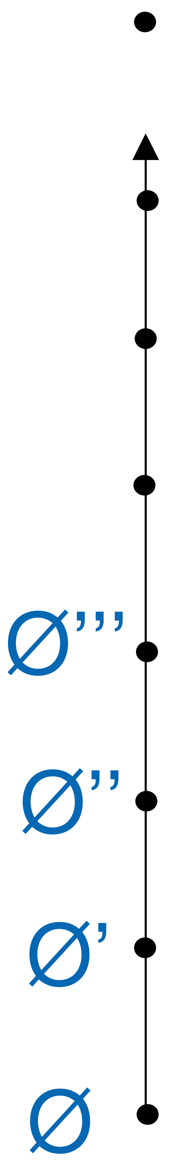
Partially Ordered Sets

HEINZ NIXDORF INSTITUTE
University of Paderborn
Algorithms and Complexity

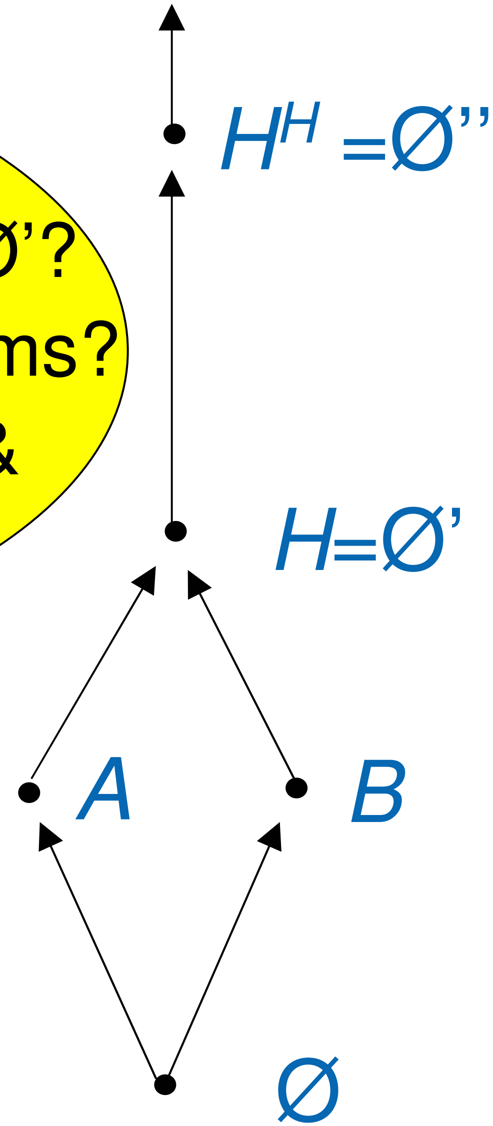


Emil Post 1944:

a) Is anything in between Ø and Ø'?

b) Are there incomparable problems?

Answered 1956/57 by Friedberg & Muchnik: such *A*,*B* exist!



Two Incomparable Problems

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Proof idea: Show there exist semidec $A,B \subseteq \mathbb{N}$ such that

To each prog. P exists $\underline{x}[P]$ s.t.: $\underline{x} \in A \Leftrightarrow P^B$ accepts $\underline{x}$

To each prog. Q exists $\underline{y}[Q]$ s.t.: $\underline{y} \in B \Leftrightarrow Q^A$ accepts $\underline{y}$

Start with $\underline{x},\underline{y}:=0$, $A,B:=\emptyset$. Enumerate all progs $P^?,Q^?$.

- If P^B accepts $\underline{x}$, set $A:=A\cup\{\underline{x}\}$; else keep A.
  Let $\underline{x} := \underline{x}+1$
- If Q^A accepts $\underline{y}$, set $B:=B\cup\{\underline{y}\}$; else keep B.
  Let $\underline{y} := \underline{y}+1$

But oracles A, B change, may later violate witness condition "$\underline{x} \in A \Rightarrow P^B$ accepts $\underline{x}$"…

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Start with $\underline{x},\underline{y}:=0$, $A,B:=\emptyset$. Enumerate all progs $P^?,Q^?$.

- If P^B accepts $\underline{x}$, set $A:=A\cup\{\underline{x}\}$
  and $\underline{y} :=$ max{ $\underline{y}$, largest oracle query by P^B on $\underline{x}$ } +1
- If Q^A accepts $\underline{y}$, set $B:=B\cup\{\underline{y}\}$
  and $\underline{x} :=$ max{ $\underline{x}$, largest oracle query by Q^A on $\underline{y}$ } +1

But oracles A, B change, may later violate
witness condition "$\underline{x}\in A \Leftarrow P^B$ accepts $\underline{x}$"…