

Let's program a calculator:

```
Welcome to KAIST SuperCalculator!
> 3 * (5 + 7 * 2) + 30 * 2 / 15
==> 61
> 110 - (23 + 12) * (15 - 12)
==> 5
```

We need four kinds of tokens:

- Number constants, such as 12 or 34.56;
- Variable names (“identifiers”), such as abc12;
- Operators (usually one-letter), such as +, *, or (;
- a stop token (end of input).

We use the following rules:

- Whitespace is skipped;
- A number is a string of digits with possibly a decimal point;
- an identifier starts with a letter or ‘_’, and consists of letters, digits, and underscores;
- anything else is a one-letter symbol token.

Tokenization means to partition the input string or text file into **tokens** (smallest meaningful units) such as numbers, identifiers, and operators.

(abc12+27 * 23.0(12abc34

Symbol: (

Identifier: abc12

Symbol: +

Number: 27.0

Symbol: *

Number: 23.0

Symbol: (

Number: 12.0

Identifier: abc34

Stop.

Whitespace (spaces, line feeds, tabs) is already removed by tokenization.

Note: Tokenizer knows nothing about the syntax of expressions or the programming language.

An **expression** is a sum (with + or -) of terms.

A **term** is a product (with * or /) of items.

An **item** is either a number, or a variable name, or an expression enclosed in parentheses.

For each syntactical element (that is, “expression”, “term”, and “item”) we write a method to parse it.

Since **parseExpression** calls **parseTerm**, **parseTerm** calls **parseItem**, and **parseItem** may call **parseExpression**, recursive descent parsing automatically leads to **indirect recursion**.

If you want to understand **indirect recursion**, please see the **next slide**.

For an explanation of **indirect recursion**, please see the previous slide.

Sine and cosine can be computed using the following identities:

$$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

$$\cos x = 1 - 2\left(\sin \frac{x}{2}\right)^2$$

Your computer uses *indirectly recursive* methods

`sin(x: Double)` and `cos(x: Double)` that compute $\sin x$ and $\cos x$ using these identities. The base case occurs when x is so small that a direct approximation is possible.